Conservative Quadratic RSM combined with Incomplete Small Composite Design and Conservative Least Squares Fitting

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A new quadratic response surface modeling method is presented. In this method, the incomplete small composite design (ISCD) is newly proposed to reduce the number of experimental runs than that of the SCD. Unlike the SCD, the proposed ISCD always gives a unique design assessed on the number of factors, although it may induce the rank-deficiency in the normal equation. Thus, the singular value decomposition (SVD) is employed to solve the normal equation. Then, the duality theory is used to newly develop the conservative least squares fitting (CONFIT) method. This can directly control the over- or the under-estimation behavior of the approximate functions. Finally, the performance of CONFIT is numerically shown by comparing its' conservativeness with that of conventional fitting method. Also, optimizing one practical design problem numerically shows the effectiveness of the sequential approximate optimization (SAO) combined with the proposed ISCD and CONFIT.

Key Words : Response Surface Model, Sequential Approximate Optimization, Incomplete Small Composite Design, Conservative Least Squares Fitting

1. Introduction

As expensive analyses and experiments are frequently encountered in the modern engineering optimizations, sequential approximate optimizations (SAO) combined with response surface models (RSM) have gained in popularity. Thus, it is important in constructing response surface models to achieve an acceptable level of accuracy while attempting to minimize the computational effort, i.e. the number of system analyses or experiments. Although increasing the number of points could improve the accuracy of the approximate model, many studies have concentrated on reducing the number of points that represent expensive analyses and experiments (Box and Wilson, 1951 ; Box and Hunter, 1957 ; Hartley, 1959 ; Westlake, 1965 ; Draper, 1985 ; Draper and Lin, 1990 ; Myers and Montgomery, 1995). Among them, although the small composite design (SCD), proposed by Draper (1985) and Draper and Lin (1990), is one of the minimum design for constructing the second-order response surfaces, it requires more than the saturate points that are heuristically determined. For this reason, response surface models, like most other experimental techniques, are severely limited in the number of design variables that they handle.

However, it is noted that the general procedure of design optimization is iterative until some convergence criteria are satisfied, even though several approximation techniques are employed (Barthelemy and Haftka, 1993). In other words, in

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order to guarantee the convergence of the iterative optimization, many approximate models are sequentially employed during the subsequent iterations. This represents that the complete quadratic response surface model is not a necessary requirement in the sequential approximate optimization process.

This study proposes an Incomplete Small Composite Design (ISCD) for efficiently constructing the quadratic response surface model and a conservative least-square fitting method (CONFIT) for improving the feasibility and the usability of response surface model in the engineering optimization. Chapter 2 reviews the composite designs for second-order response surface modeling. Especially, the small composite designs are more detailed reviewed. Chapter 3 presents the basic idea of the proposed ISCD and describes the least square methods to solve the rank-deficient normal equations caused by ISCD. Then, the basic algorithm for CONFIT is clearly explained. In chapter 4, a computational procedure of the SAO combined with the ISCD and the CONFIT. Then, in Chapter 5, the performances of the proposed methods are numerically examined by solving two practical design problems. Finally, Section 6 presents the conclusion of this study.

2. Review of Composite Designs for Second-Order Response Surface Modeling

Composite designs for fitting second-order response surfaces were first introduced by Box and Wilson (1951) and followed up by Box and Hunter (1957). A composite design, shown in Fig. 1, consists of a 2^k factorial or a 2^{k-q} fractional factorial portion, with runs selected from the 2^k runs $(x_1, x_2, \cdots, x_k)=(\pm 1, \pm 1, \cdots, \pm 1)$ usually of resolution Vor higher, plus a set of $2k$ axial points at a distance α from the origin, plus n_o center points. Thus we have a total of $2^{k-q}+2k+n_o$ points. In general, the 2^{k-q} portion (or cube) may be repeated c times and the axial points (or stars) may be repeated s times. The value of a n_o, c and s are to be selected by the experimenter.

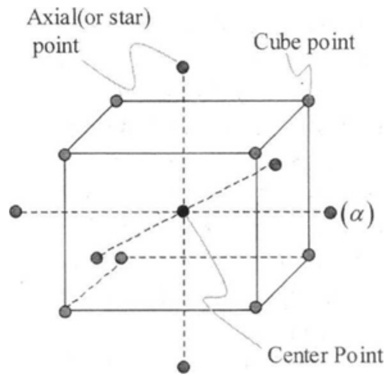


Fig. 1 A typical central composite design for $k=3$, $q=0$ and $n_0=1$

Composite designs are extremely useful for sequential experimentation in which the cube portion is used to allow estimation of the first-order effects and the later addition of the star points permits second-order terms to be added to the model and estimated. However, when experimentation is expensive, difficult, or time-consuming, small designs might be appropriate, especially when an independent estimate of experimental error is available.

Hartley (1959) pointed out that, for estimation of the second-order surface, the cube portion of the composite design need not be of resolution V. It could be of resolution as low as III, provided that two-factor interactions were not aliased with other two-factor interactions. Hartley employed a smaller fraction of the 2^k factorial than is used in the original Box-Wilson designs and so reduced the total number of points. Hartley's cubes may be designated resolution III*, meaning a design of resolution III but with no words of length four in the defining relation.

Westlake (1965) provided a method for generating composite designs based on irregular fractions of the 2^k factorial system rather than using the complete factorials or regular fractions of factorials employed by Box and Wilson (1951) and Hartley (1959). Westlake provided three examples for 22-run designs for $k=5$, one example of a 40-run design for $k=7$, and one example of a 62-run design for $k=9$.

Draper (1985, 1990) proposed an alternative approach to obtaining small composite design

(SCD), which employed columns of the Plackett-Burman designs (1946) rather than regular or irregular fractions. Draper (1985) and Draper and Lin (1990) have shown that many small composite designs exist. The formation of these designs are constructed by (1) using the $2k$ axial runs plus center runs, (2) adding the k columns of a Plackett-Burman design for the cube portion to avoid singularity or near singularity, (3) while removing one of each set of duplicates if duplicate runs exist. Draper provided, using 12-run, 28-run and 44-run Plackett-Burman designs, 22-run design for $k=5$, 42-run design for $k=7$ and 62-run design for $k=9$, respectively. However, his approach can not give a general design assessed on the number of factors, because it is an another optimization problem. For the detailed information, one may refer to the references (Draper, 1985 ; Draper and Lin, 1990).

3. Conservative and Efficient Quadratic Response Modeling Method

This study proposes an Incomplete Small Composite Design (ISCD) for large scaled response surface modeling and optimization in the context of sequential approximate optimization.

3.1 Constructing the incomplete small composite design

In the original SCD, it is very difficult to select the minimum columns of a Plackett-Burman design for the cube portion to avoid singularity or near singularity while removing one of each set of duplicates if duplicate runs exist, because it is also an optimization problem. Thus, Draper (1985, 1990) proposed only three designs assessed on the three cases such as $k=5$, $k=7$ and $k=9$.

ISCD fundamentally uses $2k$ axial runs plus center runs to represent curvatures of the system and allow for efficient estimation of the pure quadratic terms. However, for constructing the cube portion, although the Plackett-Burman design is used, only the minimum number of runs are directly used, which are available from the Plackett-Burman design for the k factor. For more detailed description, the minimum number of runs to be performed in order to assess the factors under study is listed in Table 1. Then, the total number of points in cube and star, excluding center points, in various composite designs previously discussed, are summarized in Table 2.

Table 1 Number of runs assessed on the number of factors in the Plackett-Burman design

Number of runs	Number of real factors, k	Number of columns
4	2~3	4
8	4~7	7
12	8~11	11
20	12~19	19
24	20~23	23

Table 2 Total experimental points excluding center points in some small composite designs

		Factors, k							
		3	4	5	6	7	8	9	10
	Number of coefficients	10	15	21	28	36	45	55	66
	Number of star points ($2k$)	6	8	10	12	14	16	18	20
Points For cube portion	Box and Hunter (1957)	8	16	16	32	64	64	128	128
	Hartley (1959)	4	8	—	16	32	—	64	—
	Westlake (1965)	—	—	12	—	26	—	44	—
	Draper (1985)	—	—	12	—	28	—	44	—
	Draper & Lin (1990)	—	—	11	—	22	—	38	—
	Proposed ISCD	4	8	8	8	8	12	12	12

The symbol '-' denotes that the design is not provided by the developer.

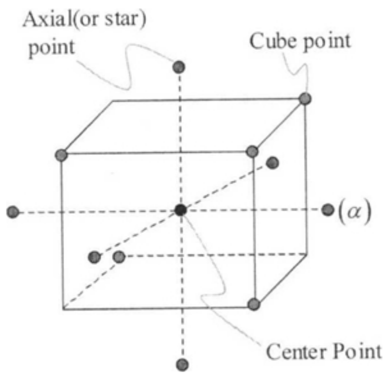


Fig. 2 The incomplete small composite design for $k=3$, $q=1$ and $n_0=1$

Fig 2 shows the proposed ISCD for $k=3$.

This study recommends that the proposed ISCD model be used only at the first iteration in the sequential approximate optimization (SAO) process. After the first iteration, the SAO gives an approximate optimum. In the next iteration, the exact function values are evaluated at this approximate optimum. Then, the approximate models are updated using the information at the pre-sampled ISCD plus this new point.

3.2 Least squares fitting based on the singular value decomposition

In order to simplify the explanation of the construction of quadratic response surface models using ISCD, consider the following matrix notation as :

$$\mathbf{y}=\mathbf{X}\boldsymbol{\beta}+\mathbf{e}, \tag{1}$$

where $\mathbf{y}$ is a vector of N observations, $\mathbf{X}$ is a matrix of known constants, $\boldsymbol{\beta}$ is a vector of n parameters to be estimated, and $\mathbf{e}$ is the vector of random errors. The matrix $\mathbf{X}=(x_{ij})$ is sometimes called the design matrix ; it has N rows and n columns.

In estimating the unknown constants, β_j, by the method of least squares, a set $\tilde{\beta}_1, \tilde{\beta}_2, \cdots, \tilde{\beta}_n$, which minimize the sum of the squares of the residuals as

$$\min_{\tilde{\beta}} \frac{1}{2}R_e=\frac{1}{2}\tilde{\boldsymbol{\beta}}^T(\mathbf{X}^T\mathbf{X})\tilde{\boldsymbol{\beta}}-(\mathbf{X}^T\mathbf{y})\tilde{\boldsymbol{\beta}}, \tag{2}$$

which can be simplified, in matrix form, as

$$(\mathbf{X}^T\mathbf{X})\tilde{\boldsymbol{\beta}}=\mathbf{X}^T\mathbf{y}. \tag{3}$$

This is called the normal equations. However, it is noted that the normal equations have a unique solution vector,

$$\tilde{\boldsymbol{\beta}}=(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}, \tag{4}$$

only if the n columns of $\mathbf{X}$ are linear independents.

As you can see, the proposed ISCD can give a rank-deficient $\mathbf{X}$ matrix because its number of points is less than n. Hence, this study uses a Singular-Value Decomposition (SVD) method (Press et. Al., 1986) to solve the normal equation of Eq. (3), because, in case of an over-determined system, it produces a solution that is he best approximation in the least-square sense, and in case of an under-determined system, it produces a solution whose values (for us, the $\tilde{\boldsymbol{\beta}}$) are smallest in the least square sense. SVD method decompose the square matrix $\mathbf{X}^T\mathbf{X}$ as

$$\mathbf{X}^T\mathbf{X}=\mathbf{U}\cdot\mathbf{W}\cdot\mathbf{V}^T, \tag{5}$$

where the left singular vectors $\mathbf{U}$ are an $n\times n$ column-orthogonal matrix, the right singular vectors $\mathbf{V}$ are $n\times n$ orthogonal matrix, and an $n\times n$ diagonal matrix $\mathbf{W}$ contains singular values. As $\mathbf{U}$ and $\mathbf{V}$ are orthogonal respectively, their inverses are equal to their transposes. Also, $\mathbf{W}$ is diagonal, thus its inverse is the diagonal matrix whose elements are the reciprocals of the elements w_j. Hence, the inverse of $\mathbf{X}^T\mathbf{X}$ is

$$(\mathbf{X}^T\mathbf{X})^{-1}=\mathbf{V}\cdot[\mathrm{diag}(1/w_j)]\cdot\mathbf{U}^T \tag{6}$$

The only thing that can go wrong with this construction is for one of the singular values w_j's to be zero or for it to be so small that its value is dominated by round-off error and therefore unknowable. In general, to overcome this problem, if any singular value w_i is zero or less than $\varepsilon\cdot w^{\max}$, SVD sets its reciprocal in Eq. (5) to zero, not to infinity. The value of $w^{\max}$ denotes the maximum value of w_j, $j=1, 2, \cdots, n$. This corresponds to adding to the fitted parameters $\tilde{\boldsymbol{\beta}}$ a zero multiple, rather than some random large multiple, of any linear combination of basic functions $[\mathbf{U}_i\cdot(\mathbf{X}^T\mathbf{y})]\mathbf{v}_i$ that are degenerate in the fit. In this study, the small value ε is recommended as $\varepsilon=1\times10^{-5}$.

3.3 Conservative least squares fitting based on the duality theory

In section 3.2, we explain SVD for least squares fitting. In this section, the conservative least square method is presented. As you can see, the approximate function, obtained from the conventional least square fitting, passes through the observations shown in Fig. 3. Although this can be effective to understand the trend of the observations throughout the approximated range, it can cause a serious problem in the convergence of the sequential approximate optimization process. In other words, a feasible solution obtained from the approximate optimization is not feasible in the real design space in spite of successive updating the approximate models. This can retard the convergence rate of the approximate optimizations or fail to converge. Thus, this study proposes the conservative least squares fitting method (CONFIT) shown in Fig. 4.

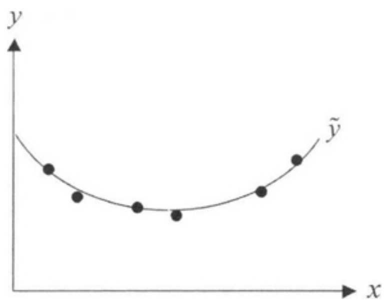
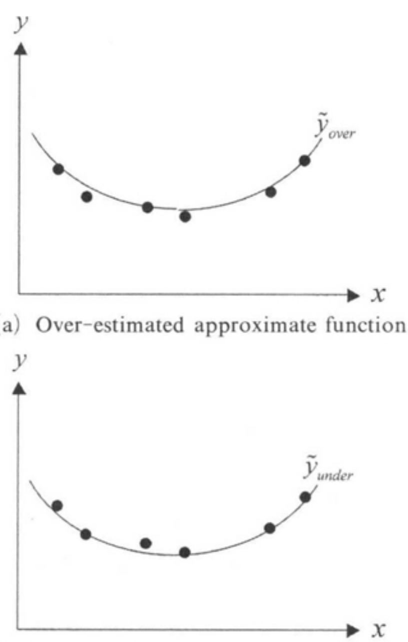


Fig. 3 The approximate function $\tilde{y}$ obtained from the conventional least square fitting method



(a) Over-estimated approximate function

(b) Under-estimated approximate function

Fig. 4 The approximate function $\tilde{y}$ obtained from the conservative least square fitting method

After the least square fitting described in section 3.2, the approximate observations are evaluated as $\tilde{\mathbf{y}}=\mathbf{X}\boldsymbol{\beta}$. Then, select the violated sets, $\mathbf{S}_o=\{i: \tilde{y}_i<y_i,\ i=1, \cdots, k\}$ for the over-estimated approximate function $\tilde{y}_{over}$ and $\mathbf{S}_u=\{i: \tilde{y}_i>y_i,\ i=1, \cdots, k\}$ for the under-estimated approximate function $\tilde{y}_{under}$, respectively. Hence, the formulation of conservative least square fitting finds $\tilde{\boldsymbol{\beta}}_c$ to minimize

$$\frac{1}{2}\boldsymbol{\beta}_c^T(\mathbf{X}^T\mathbf{X})\tilde{\boldsymbol{\beta}}_c-(\mathbf{X}^T\mathbf{y})\tilde{\boldsymbol{\beta}}_c \tag{7}$$

while satisfying

$$\mathbf{X}_a\tilde{\boldsymbol{\beta}}_c=\mathbf{y}_a, \tag{8}$$

where the subscript a denotes that their components are included in the violated set $\mathbf{S}_o$ or $\mathbf{S}_u$. Using the Wolfe dual method (Fletcher, 1987), Eqs. (7) and (8) can be transformed as

$$\max_{\tilde{\beta}_c,\lambda}\frac{1}{2}\tilde{\boldsymbol{\beta}}_c^T(\mathbf{X}^T\mathbf{X})\tilde{\boldsymbol{\beta}}_c-(\mathbf{X}^T\mathbf{y})^T\tilde{\boldsymbol{\beta}}_c+\boldsymbol{\lambda}^T(\mathbf{X}_a\tilde{\boldsymbol{\beta}}_c-\mathbf{y}_a) \tag{9}$$

subject to

$$(\mathbf{X}^T\mathbf{X})\tilde{\boldsymbol{\beta}}_c-(\mathbf{X}^T\mathbf{y})+\mathbf{X}_a\boldsymbol{\lambda}=\mathbf{0}, \tag{10}$$

where $\boldsymbol{\lambda}$ is the Lagrange multiplier vector. Using Eq. (10) to eliminate $\tilde{\boldsymbol{\beta}}_c$ from the dual objective function of (9) gives the simplified problem as

$$\max_{\lambda}-\frac{1}{2}\boldsymbol{\lambda}^T\mathbf{A}\boldsymbol{\lambda}+\mathbf{c}^T\boldsymbol{\lambda}, \tag{11}$$

where $\mathbf{A}=\mathbf{X}_a(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}_a^T$ and $\mathbf{c}=\mathbf{X}_a(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}-\mathbf{y}_a$. In Eq. (11), the constant terms are neglected. Then, the optimum dual variables $\boldsymbol{\lambda}^*=\mathbf{A}^{-1}\mathbf{c}$ can be obtained from Eq. (11). Hence, substituting this $\boldsymbol{\lambda}^*$ into Eq. (10) gives the explicit solution of the unknown coefficients $\tilde{\boldsymbol{\beta}}_c$ for conservative fitting as

$$\tilde{\boldsymbol{\beta}}_c=(\mathbf{X}^T\mathbf{X})^{-1}(\mathbf{X}^T\mathbf{y}-\mathbf{X}_a^T\boldsymbol{\lambda}^*) \tag{12}$$

In the above evaluations, $(\mathbf{X}^T\mathbf{X})^{-1}$ is directly used from Eq. (6) and $\mathbf{A}^{-1}$ is computed by SVD in the same way that evaluates $(\mathbf{X}^T\mathbf{X})^{-1}$ in Eq. (6). For clear description, the conservativeness of the proposed method is numerically examined in Section 5.1. In the next comparison, the central composite design (CCD) is used only for more clearly showing the conservativeness of the CONFIT.

4. Sequential Approximate Optimization Combined with ISCD and CONFIT

In order to use the ISCD and the CONFIT in the sequential approximate optimization (SAO), The following computational procedure is described as :

Step 0. Given the design range Γ_d, the convergence tolerances ε_1 and ε_2. Set $t=0$ and the design set $\mathbf{D}_t$ whose number of sampling points is one center point plus the points in cube and star listed in Table 2 (ISCD).

Step 1. Evaluate function values of objective $f(\mathbf{x})$ and inequality constraint functions $g_j(\mathbf{x})$, $j=1, \cdots, m$ for the sampling po ints in $\mathbf{D}_t$ and store them into the set $\mathbf{F}_t$ and $\mathbf{G}_t$, respectively.

Step 2. Construct the approximate functions $\tilde{f}(\mathbf{b})$ and $\tilde{g}_j(\mathbf{x})$, $j=1, \cdots, m$ using the CONFIT such as equations (4) and (12).

Step 3. Solve the following approximate optimization problems : minimize $\tilde{f}(\mathbf{x})$ subject to $\tilde{g}_j(\mathbf{x}) \leq 0, j=1, \cdots, m$ and $x_i^L \leq x_i \leq x_i^U$ for $i=1, \cdots, n$. Let $\tilde{\mathbf{x}}_i^*$ be the approximate optimum. Go to *Step 4*.

Step 4. Evaluate the exact function values at the approximate optimum $\tilde{\mathbf{x}}_i^*$. If the convergence criteria of $|f(\tilde{\mathbf{x}}_i^*)-f(\mathbf{x}_t)| \leq \varepsilon_1 | f(\mathbf{x}_t)|$ and $g_j(\tilde{\mathbf{x}}_i^*) \leq \varepsilon_2$ for $j=1, \cdots, m$ are satisfied for consecutive iterations, then the optimization is terminated. Otherwise, go to *Step 5*.

Step 5. Update the design set $\mathbf{D}_t$ and function value sets $\mathbf{F}_t$ and $\mathbf{G}_t$ by including $\tilde{\mathbf{x}}_i^*$ and its' corresponding function values. Then, Return to *Step 2* with $t=t+1$.

In Step 3, the approximate optimization problem can be solved using any constrained optimization algorithms (Vanderplaats, 1984). Among them, this study uses the augmented Lagrange multiplier method (Kim, 2002).

5. Numerical Studies

Now, we will show the numerical performance of the proposed ISCD and CONFIT, we will examine the conservativeness of the CONFIT using central composite design (CCD)' points. Then, the numerical performance of the SAO, developed based on the computational procedures described in section 4, is examined by solving the tracked vehicle suspension design problem. In this SAO, the values of convergence tolerances are used as $\varepsilon_1=1\times10^{-2}$ and $\varepsilon_2=1\times10^{-4}$.

5.1 Response surface modeling for sled test results

In order to simply estimate the performance of the occupant protection in the vehicle, sled test is widely used. In this study, three design variables are chosen as 1) the airbag vent hole size, 2) the seat belt strain rate and 3) the airbag firing time. The performance indexes to be approximated are the head injury criterion (HIC) and the chest acceleration of the dummy. For constructing the

Table 3 The sled test simulation results for the central composite design

Runs	x_1	x_2	x_3	HIC	Chest Accel.
1	−2.6	−1.5	−2.7	783.06	48.362
2	2.4	−1.5	−2.7	697.33	47.804
3	−2.6	1.5	−2.7	813.28	47.947
4	2.4	1.5	−2.7	702.08	47.366
5	−2.6	−1.5	2.7	860.05	48.354
6	2.4	−1.5	2.7	751.18	48.162
7	−2.6	1.5	2.7	889.19	47.941
8	2.4	1.5	2.7	768.77	46.436
9	−4.4	0.0	0.0	832.31	47.807
10	4.0	0.0	0.0	671.89	46.806
11	0.0	−2.5	0.0	736.99	48.891
12	0.0	2.5	0.0	788.37	46.067
13	0.0	0.0	−4.5	735.08	48.830
14	0.0	0.0	4.5	842.50	48.109
15	0.0	0.0	0.0	754.35	44.461

Table 4 The approximate functions obtained by both least squares fitting methods

Conventional Least Square Fitting	$\tilde{H}(\mathbf{x}) = 751.75 - 20.35x_1 + 8.158x_2 + 12.327x_3 + 0.1992x_1^2 + 2.972x_2^2 + 2.206x_3^2 + -.2105x_1x_2 - 0.6036x_1x_3 + 0.329x_2x_3$
	$\tilde{C}(\mathbf{x}) = 44.519 - 0.0898x_1 - 0.3813x_2 - 0.0494x_3 + 0.150x_1^2 + 0.4556x_2^2 + 0.1895x_3^2 + -0.0465x_1x_2 - 0.0106x_1x_3 - 0.0396x_2x_3$
Conservative Least Square Fitting	$\tilde{H}(\mathbf{x}) = 754.35 - 20.53x_1 + 7.8913x_2 + 11.929x_3 + 0.42494x_1^2 + 3.6433x_2^2 + 2.4139x_3^2 + -0.77x_1x_2 - 0.85704x_1x_3 - 0.00667x_2x_3$
	$\tilde{C}(\mathbf{x}) = 44.643 - 0.07044x_1 - 0.44674x_2 - 0.0665x_3 + 0.15278x_1^2 + 0.5009x_2^2 + 0.1919x_3^2 + -0.0734x_1x_2 - 0.0265x_1x_3 - 0.0167x_2x_3$

accurate quadratic response surface modeling, a central composite design (CCD) is used. Thus, the total number of experimental points is $15(1+2 \cdot 3+2^3)$. Table 3 lists the simulation results side by side. In this table, the values of design variables are the deviation from the nominal values. Two least squares fitting methods such as Eq. (4) and Eq. (12) are used to construct the surrogate models. As those two performance indexes are preferred to minimize, the over-estimated approximation function is employed in the CONFIT.

Table 4 lists these approximate functions. Symbols $\tilde{H}(x)$ and $\tilde{C}(x)$ represent the approximate performance of HIC and chest acceleration of the dummy, respectively. It is noted that both fitting methods give somewhat different approximate functions. In other words, in order to improve the conservativeness, CONFIT does not simply shift the constant term of the approximate function obtained by the conventional fitting method but change whole the coefficients of its' approximate function.

Now, in order to examine the conservativeness of the CONFIT, the relative deviations are checked at the 15 experimental points. These deviations are defined as $(\tilde{H}(x) - H(x))/H(x)$ and $(\tilde{C}(x) - C(x))/C(x)$, respectively. Fig. 5 shows them graphically. As you can see, the proposed CONFIT gives positive- or zero-value deviations at all the experimental points. The conventional method, however, gives positive- or negative-value deviations. That is, the conventional method gives the over-estimated results in some design region and the under-estimated results in the other design region. We believe that these irregular estimations of the conventional fitting method make the convergence of sequential approximate optimization to be difficult.

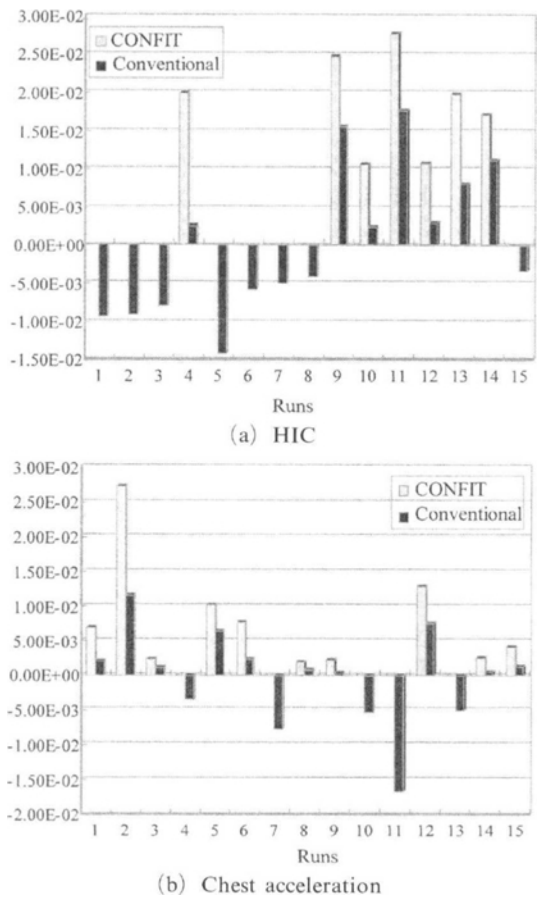


Fig. 5 The comparisons of the relative deviations for two fitting methods

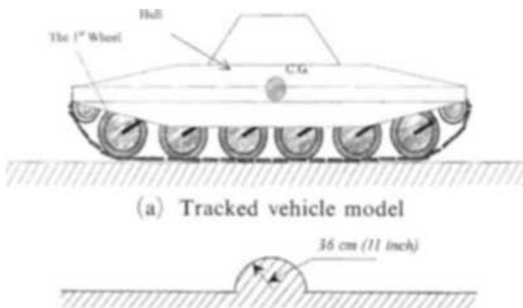
Table 5 The optimization results of a tracked vehicle system design

	Lower Bound	Upper Bound	Initial Design	Final Design
x_1	175	185	180	180.17
x_2	140	160	150	150.05
x_3	130	150	140	143.59
x_4	140	160	150	151.34
x_5	40000	50000	45000	40000
x_6	0.13	0.14	0.135	0.139
x_7	110	120	115	115.24
x_8	0.0035	0.0042	0.00385	0.0039
x_9	500	700	600	584.31
Objective			23.106	15.071
Max. of constraint			−0.0475	−0.0012
No. of analyses			—	34

5.2 Tracked vehicle suspension design

Fig. 6(a) shows a tracked vehicle suspension system, which is to be designed to minimize the extreme acceleration of the mass center when the vehicle run over a bump (36 cm) shown in Fig. 6 (b) for a given speed (40 km/h). The tracked vehicle model is composed of a hull, two sprockets, six wheels with HSU suspension systems and track. 9 design variables are divided the following three groups: 1) the charging pressures for the HSU systems of the 1st, 2nd, 5th and 6th wheels, 2) the static track tension and 3) the length of a gas chamber, the pre-load on Bellevile springs, the diameter of orifices, and the choking flow rate for HSU systems. The motion of the vehicle is constrained so that the maximum acceleration of mass center, wheel travels for the six wheels, and static wheel loads for the six wheels are within given limits. Also, the charging pressures of HSU systems for the 3rd and 4th wheels are within given limits. The Recurdyn 1.0 (Bae et. al. 1999; Han et. al. 1999) is utilized to model and perform the multi-body system dynamic analyses.

Table 5 lists the initial and final designs side by side. As you can see, the saturated design for the 9 variables requires 55 points for constructing a quadratic response surface model. However, the proposed ISCD uses only the 31 points such as the 19 points for the linear and pure quadratic terms and the 12 points from the Placket-Burman design for the linear and two-factor interaction terms. Then, only 3 points are sequentially added



(a) Tracked vehicle model

(b) Single bump

Fig. 6 The seven degree-of-freedom tracked vehicle suspension system

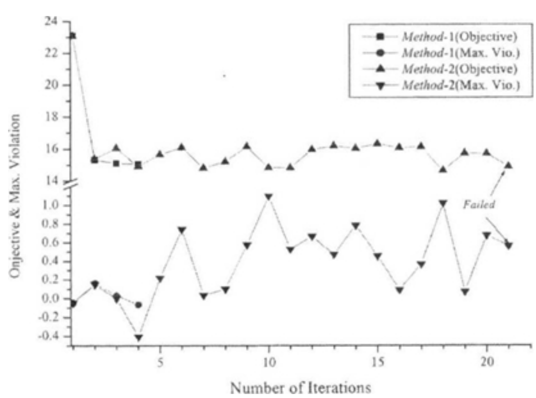


Fig. 7 The convergence histories of two approximate optimizations

as the approximate optimization progresses. Thus, total 34 analyses is used to solve this design problem.

Fig. 7 shows the convergence histories of the two approximate optimization methods. In them, the Method-1 denotes the ISCD combined with CONFIT and the Method-2 denotes the ISCD combined with conventional least squares fitting method. In the Method-1, the over-estimation is employed for approximating the object and constraint functions. It is noted that the Method-2 is oscillated during optimization processes and finally failed to convergence. We believe that you can see these oscillations whenever the response surface models approximated by the conventional fitting method is used in the SAO.

6. Concluding Remarks

In order to construct a conservative and economical quadratic response surface model, this study proposes the incomplete small composite design (ISCD) and the conservative least squares fitting (CONFIT) method. As the original SCD is a heuristic approach, one can not obtain a unique design assessed on the number of variables. However, the proposed ISCD gives a unique and economic design table, although it may induce the rank-deficiency in the normal equation. Thus, the singular value decomposition (SVD) is used to solve the rank-deficient normal equations. Also, in order to overcome the oscillation phenomena and the convergence difficulty in the sequential approximate optimization combined with conventional least squares fitting method, the conservative least squares fitting (CONFIT) method is newly proposed, which is based on the duality theory and SVD.

In order to show the numerical performance of the proposed methods, a general least squares fitting program implementing the CONFIT and the conventional method is developed. Also, a sequential approximate optimization program combined with ISCD and CONFIT is developed. Then, the fitting program is used to construct the surrogate model for approximating the sled test results and the SAO program is employed for solving the tracked vehicle suspension system design. In these numerical tests, it is shown that the CONFIT can successfully construct more conservative RSM than that of conventional method. Especially, in the tracked vehicle suspension system design, it is noted that the CONFIT can play an important role in the convergence of SAO. We believe that the CONFIT is valuable, even though it may be combined with other experimental designs.

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